 (MUM.D(I) – MJC – 1)

Degree (Sem – I) Examination – 2023

Session – (2023-27)

MATHEMATICS

(Modal Question – 2)

PAPER (MJC – 1)

Time: 3 hrs. **Full Marks - 70**

Candidates are required to give their answers in their own words as far as practicable.

Figures in the margin indicate full marks.

Answer from all Groups as directed.

Group – A

1. Choose the correct answer of the following:

$(2 \times 10 = 20)$

(a) The sum of the series $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots \infty =$

(i) $\pi^2/3$
(ii) $\pi^2/4$
(iii) $\boldsymbol{\pi^2/6}$
(iv) $\pi^2/8$

(b) The value of $\sinh 0 =$

(i) 0
(ii) 1
(iii) ∞
(iv) None of these

(c) The set of all real functions defined on the closed unit interval $[0, 1]$ has cardinal number

(i) $\boldsymbol{c}$
(ii) 2^c
(iii) $2c$
(iv) λ_0

(d) The function and relation are such that

(i) Every relation is a function.
(ii) Every function is a relation.
(iii) No function is a relation.
(iv) No relation is a function.

(e) If d is the greatest common divisor of b and c, then there exist integers x_0 and y_0 such that

(i) $d = (b + c)x_0 + (b - c)y_0$
(ii) $d = (b + c)x_0 - (b - c)y_0$
(iii) $\boldsymbol{d = bx_0 + cy_0}$
(iv) All of the above.

(f) Euclid's algorithm is used for finding

(i) **GCD of two numbers.**
(ii) GCD of more than three numbers.
(iii) LCM of two numbers.
(iv) LCM of more than two numbers.

(g) The rank of the matrix $\begin{bmatrix} -1 & -2 & 1 \\ 1 & 0 & 5 \end{bmatrix}$ is

(i) 1
(ii) 2
(iii) **3**
(iv) 0

(h) For what value of λ, do the simultaneous equation $2x + 3y = 1, 4x + 6y = \lambda$ have infinite solutions?

(i) $\lambda = 0$
(ii) $\lambda = 1$
(iii) $\boldsymbol{\lambda = 2}$
(iv) $\lambda \neq 2$

(i) If the equation $a_0x^3 + 3a_1x^2 + 3a_2x + a_3 = 0$ is transformed by the substitution $z = a_0x + a_1$ to the form $z^3 + 3Hz + G = 0$, then H is equal to:

(i) $\boldsymbol{a_0a_2 - a_1^2}$
(ii) $a_0a_1 - a_2^2$
(iii) $a_1a_2 - a_0^2$
(iv) None of these.

(j) If $q > 0, r > 0$, then the cubic $x^3 + qx + r = 0$ has:

(i) All roots real and positive.
(ii) All roots real and negative.
(iii) One positive real root and two imaginary roots.
(iv) One negative real root and two imaginary roots.

Group – B

Answer **any four** questions of the following:

$(5 \times 4 = 20)$

2. Find the equation whose roots are the p th powers of the roots of the equation $x^2 - 2x\cos\theta + 1 = 0$.
3. Define equivalence relation on a set and prove that the relation 'congruence modulo 5' is an equivalence relation on the of integers.
4. If $f: X \to Y$ be a mapping and let $A, B \subseteq Y$, then show that $f^{-1}(A \cap B) = f^{-1}(A) \cap f^{-1}(B)$.
5. If d is common divisor of a and b and $g = \gcd(a, b)$, then prove that $d \mid g$.
6. Find the rank of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 0 & 5 & -10 \end{bmatrix}.$$

7. If α, β, γ be the roots of the equation $x^3 + px^2 + qx + r = 0$, find the value of

$$(\beta + \gamma - \alpha)^2 + (\gamma + \alpha - \beta)^2 + (\alpha + \beta - \gamma)^2.$$

Group – C

Answer **any three** questions of the following:

$(10 \times 3 = 30)$

8. Find the sum of a series of cosines of n angles, which are in arithmetical progression.
9. State and prove Cantor's theorem.
10. State and prove Fundamental Theorem of Arithmetic.
11. Prove that the transpose of a matrix is the same as that of the original matrix, hence find the rank of the matrix
$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$$
12. Solve the equation $3x^4 - 25x^3 + 50x^2 - 50x + 12 = 0$ having given that the product of two roots is 2.