 (MUM.D(I) – MIC – 1)

Degree (Sem – I) Examination – 2023

Session – (2023-27)

MATHEMATICS

(Modal Question – 1)

PAPER (MIC – 1)

Time: 3 hrs. **Full Marks - 70**

Candidates are required to give their answers in their own words as far as practicable.

Figures in the margin indicate full marks.

Answer from all Groups as directed.

Group – A

1. Choose the correct answer of the following:

$(2 \times 10 = 20)$

(a) If $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin\theta$, then value of n is:

(i) Positive or negative integers.

(ii) Rational numbers.

(iii) Both (i) and (ii).

(iv) None of the above.

(b) $\sinh^2 x - \cosh^2 x = \_\_$

(i) 1

(ii) -1

(iii) 0

(iv) 2

(c) The period of e^z is:

(i) π

(ii) 2π

(iii) πi

(iv) $2\pi i$

(d) Let $f: R \to R$, where R is the set of real numbers defined by $f(x) = x^2$, then $f^{-1}(4)$ is equal to:

(i) $\{2\}$

(ii) $\{-2\}$

(iii) $\{-2, 2\}$

(iv) $\{4\}$

(e) The set R of all real numbers in the interval $[0, 1]$ is:

(i) Countable

(ii) Uncountable

(iii) Denumerable

(iv) None of these

(f) The maximum number of equivalence relations on the set $A = \{1, 2, 3\}$ are:

(i) 5

(ii) 8

(iii) 15

(iv) None of these

(g) If $A = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$, whenever $A^2 = B$, the value of α is:

(i) -1

(ii) 1

(iii) 4

(iv) No real value of $\boldsymbol{\alpha}$.

(h) If A is a symmetric matrix, then $adj(A)$ is also:

(i) Symmetric

(ii) Skew-symmetric

(iii) Hermitian

(iv) Skew-Hermitian

(i) Every polynomial equation of an odd degree has:

(i) No real roots

(ii) At least one real root

(iii) All roots real

(iv) All roots imaginary

(j) If $\alpha, \beta, \gamma, \delta$ are the roots of the equation $a_0x^4 - 4a_1x^3 + 6a_2x^2 - 4a_3x + a_4 = 0$, then $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} + \frac{1}{\delta} =$ ___

(i) $4a_1/a_0$

(ii) $4a_3/a_0$

(iii) $\boldsymbol{4a_3/a_4}$

(iv) $4a_1/a_4$

Group – B

Answer **any four** questions of the following:

$(5 \times 4 = 20)$

2. Show that the solution of the equations $(1 + x)^2 + (1 - x)^2 = 0$ are given by $x = \pm i \tan \pi/4$.
3. Show that if $A \subseteq B$, then $A \times A = (A \times B) \cap (B \times A)$.
4. Prove that the function $f: A \to B$ given by $f(x) = x^2 + x + 1$, for all $x \in R$ is not injective.
5. If A is a Hermitian matrix, show that iA is skew-Hermitian.
6. Solve the equation $6x^3 - 11x^2 + 6x - 1 = 0$ whose roots are H. P.
7. Apply Descartes's rule of signs to discuss the nature of roots of the equation $x^4 + 15x^2 + 7x - 11 = 0$.

Group – C

Answer **any three** questions of the following:

$(10 \times 3 = 30)$

8. Separate real and imaginary parts of the expression $(\alpha + i\beta)^{x+iy}$.
9. If A and B are countable sets, then prove that $A \times B$ is also countable.
10. If A and B are conformable for the product AB and B and C are conformable for the product BC, then show that $(AB)C = A(BC)$.

11. Examine the consistency and solve the following system of equations:

$$x + 2y - z = 3$$
$$3x - y + 2z = 1$$
$$2x - 2y + 3z = 2$$
$$x - y + z = -1$$

12. Solve the equation $x^3 - 9x + 28 = 0$ by Cardon's method.