**Copyright Reserved** **(MUM.D(I) – MDC – 1)**

# Degree (Sem – I) Examination – 2023

**Session – (2023-27)**

# MATHEMATICS

## (Modal Question – 1)

## PAPER (MDC – 1)

**Time: 3 hrs.** **Full Marks - 70**

**Candidates are required to give their answers in their own words as far as practicable.**

**Figures in the margin indicate full marks.**

**Answer from all Groups as directed.**

## Group – A

1. Choose the correct answer of the following:

$(2 \times 10 = 20)$

(a) The value of $(2+2i)^6 = -$:

(i) $512i$

(ii) $-515i$

**(iii)** $\boldsymbol{-512i}$

(iv) $-312i$

(b) If $|x| \leq 1$, then $x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots$ to $\infty$ is:

(i) $\tan^{-1}\frac{1+x}{1-x}$

**(ii)** $\boldsymbol{\tan^{-1} x}$

(iii) $\pi/4$

(iv) None of these

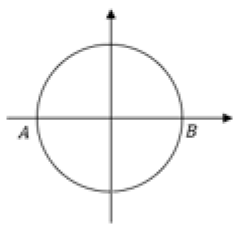
(c) $\cosh x$ is equal to:

**(i)** $\boldsymbol{(e^x + e^{-x})/2}$ (ii) $(e^x - e^{-x})/2$

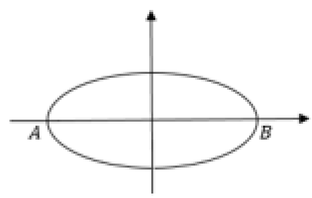
(iii) $(e^x + e^{-x})$ (iv) $(e^x - e^{-x})$

(d) Determine which of the following graph is a function in the domain from the point $A$ to the point $B$:

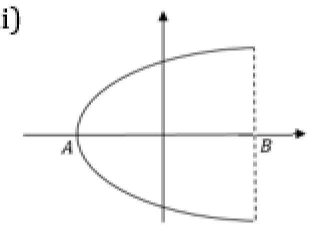
(i)



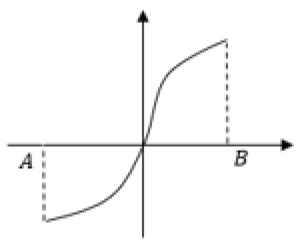

(ii)




(iii)




**(iv)**



(e) Let $R$ be a relation in a set $A$. If $R$ is reflexive, antisymmetric and transitive, then $R$ is called:
   (i) An equivalence relation in $A$.
   (ii) A partial ordered set.
   (iii) A total order relation.
   **(iv) A partial order relation in $A$.**

(f) If $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, then $A^2$ is equal to:
   (i) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
   (ii) $\begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}$
   **(iii)** $\begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{bmatrix}$
   (iv) $\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$

(g) If $A$ be a non-singular square matrix, then which one of the following is not correct:
   (i) $A$ is symmetric, then $A^{-1}$ is symmetric.
   **(ii) $A$ is symmetric, then $A^{-1}$ is skew-symmetric.**
   (iii) $(A')^{-1} = (A^{-1})'$
   (iv) All of the above.

(h) If $A^2 - A + I = 0$, then the inverse of $A$ is:
   (i) $A$
   **(ii)** $\boldsymbol{I - A}$
   (iii) $A - I$
   (iv) $A + I$

(i) How many solutions will the equation $f(x) = 10x^7 - 5x^3 + 3x - 1 = 0$ have?
   **(i) 7**
   (ii) 3
   (iii) 1
   (iv) No solution

(j) If $\alpha, \beta, \gamma$ are the roots of the equation $9 + 4x + 6x^2 - 3x^3 = 0$, then $\sum \alpha$ is:
   (i) $-4/9$
   (ii) 0
   **(iii) 2**
   (iv) 4/9

## Group – B

Answer **any four** questions of the following:

$(5 \times 4 = 20)$

2. Find the locus of the points satisfying the inequality $|z - 1| \geq 3$.
3. Assuming that $\sin x = \frac{e^{ix} - e^{-ix}}{2i}$ and $\cos x = \frac{e^{ix} + e^{-ix}}{2}$, prove that $\sin^2 x + \cos^2 x = 1$, for all values of $x$ and $y$, real or complex.
4. Give an example of a relation which is reflexive and symmetric but not transitive.

5. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, prove that $A^2 - 4A - 5I = 0$.
6. Find the inverse of the matrix $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$.
7. Find the condition that the roots of the equation $ax^2 + 3bx^2 + 3cx + d = 0$ are in arithmetic progression.

## Group – C

Answer **any three** questions of the following:

$(10 \times 3 = 30)$

8. State and prove Gregory series.
9. Let $A$ be a non-empty set and $R$ be an equivalence relation in $A$. Suppose that $a, b \in A$ are arbitrary elements, then show that either $[a] = [b]$ or $[a] \cap [b] = \phi$, where $[a]$ and $[b]$ are equivalence classes of $a$ and $b$ respectively.
10. Show that every square matrix is uniquely expressible as the sum of symmetric and skew-symmetric matrices.
11. Show that the equations

$$x + y + z = 6$$
$$x + 2y + 3z = 14$$
$$x + 4y + 7z = 30$$

are consistent and solve them.

12. (a) Find the equation whose roots are the roots of $x^5 + 4x^3 - x^2 + 11 = 0$ each diminished by 2. (6 marks)

(b) Find the least possible number of imaginary roots of the equation $x^9 - x^5 + x^4 + x^2 + 1 = 0$. (4 marks)