

Degree (Sem - I) Examination - 2023

Session - (2023-27)

MATHEMATICS

(Modal Question - 1)

PAPER (MJC - 1)

Time: 3 hrs.

Full Marks - 70

Candidates are required to give their answers in their own words as far as practicable.

Figures in the margin indicate full marks.

Answer from all Groups as directed.

Group - A

1. Choose the correct answer of the following:

(2 × 10 = 20)

(a) The n th roots of unity are in

(i) H. P.

(ii) G. P.

(iii) A. P.

(iv) None of these

(b) Which of the following formulae is correct?

(i) $\sinh z = -i \sin iz$

(ii) $\cosh z = -\cos iz$

(iii) $\tanh z = i \tan iz$

(iv) None of these

(c) If $f: R \rightarrow R$ and $g: R \rightarrow R$ are defined as $f(x) = 2x + 1$ and $g(x) = x^2 - 2$, then

$(g \circ f)(x) =$

(i) $x^2 + x + 1$

(ii) $4x^2 + x - 1$

(iii) $x^2 + 4x + 1$

(iv) $4x^2 + 4x - 1$

(d) If there is a bijection between two sets A and B , then

(i) Cardinality of A is greater than B .

(ii) Cardinality of B is greater than A .

(iii) Cardinality of B is equal to A .

(iv) None of these.

(e) If $a|b$ and $a|c$, then

(i) $a|(b + c)$

(ii) $a|(b - c)$

(iii) $a|(bx + cy)$, for any two integers x and y .

(iv) All of the above.

(f) $(n^2 + n)$ is _____ for all $n \in N$.

(i) Even

(ii) Odd

- (iii) Either even or odd
 (iv) None of these.
- (g) If the matrix A is both symmetric and skew symmetric, then
 (i) A is a diagonal matrix.
(ii) A is a zero matrix.
 (iii) A is a square matrix.
 (iv) None of these.
- (h) _____ is equal to the maximum number of linearly independent row vectors in a matrix.
 (i) Row matrix.
(ii) Rank of the matrix
 (iii) Linear matrix
 (iv) All of the above.
- (i) If the roots of the equation $2x^3 - 15x^2 + 37x - 30 = 0$ are in A. P., then roots are
 (i) 3, 2, 5/2
 (ii) 1, 3, 5
(iii) 3, 5/2, 2
 (iv) 2, 3, 5/2.
- (j) If $x = 2$ is a root is a root of $x^3 - 7x + 6 = 0$, then the other roots are
(i) -3, 1
 (ii) 3, 1
 (iii) 3, -1
 (iv) -3, -1

Group - B

Answer **any four** questions of the following:

(5 × 4 = 20)

2. Find the value of the series

$$1 - \frac{2}{3!} + \frac{3}{5!} - \frac{4}{7!} + \dots$$

3. Show that

$$\tan \left\{ i \log \left(\frac{x - iy}{x + iy} \right) \right\} = \frac{2xy}{x^2 - y^2}.$$

4. If $f: A \rightarrow B$ is one-one onto function and g is the inverse of f , then show that $f \circ g = I_B$ and $g \circ f = I_A$, where I_A and I_B are the identity mappings of A and B respectively.
 5. If $99 \mid 5544$ and $99 \mid 594$, show that $99 \mid (554x + 594y)$ for any integers x and y .
 6. Find the rank of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 0 & 5 & -10 \end{bmatrix}.$$

7. If $\alpha + i\beta$ ($\beta \neq 0$) be a root of the equation $x^3 + qx + r = 0$, then prove that 2α is a root of the equation $x^3 + qx - r = 0$.

Group - C

Answer **any three** questions of the following:

(10 × 3 = 30)

8. State and prove D' Moivre's theorem.
 9. Show that the set of rational number is denumerable.
 10. State and prove division algorithm.
 11. Prove that any square matrix can be expressed uniquely expressed as the sum of a symmetric and skew-symmetric matrix.
 12. Solve the equation $x^3 - 15x^2 - 33x + 847 = 0$ by Cardon's Method.
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